

## Techniques for Mitigating the Effect of Carrier Frequency Offset in OFDM

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**Abstract :** CFO (Carrier Frequency Offset) is a major contributor to the ICI (Inter Carrier interference) in OFDM system. ICI destroys the orthogonality between the subcarriers. Frequency synchronization in OFDM must be precise because of the narrowness of the subcarriers used. Basically to get a good performance of OFDM, the CFO should be estimated and compensated. In this paper, the effects of CFO on the SNR in an OFDM system are studied. CFO estimation algorithms are reviewed and discussed. The effect of modifications in the system parameters on estimating and compensating the effect of CFO is analyzed.

**Keywords -** CFO(Carrier frequency offset), CP(Cyclic Prefix),

### I. Introduction

The OFDM (Orthogonal Frequency Division Multiplexing) is a special case of multicarrier modulation, where a single data stream is transmitted over a number of lower rate subcarriers and these subcarriers are orthogonal to each other. As all subcarriers are of the finite duration T, the spectrum of the OFDM signal can be considered as the sum of the frequency shifted sinc functions in the frequency domain.

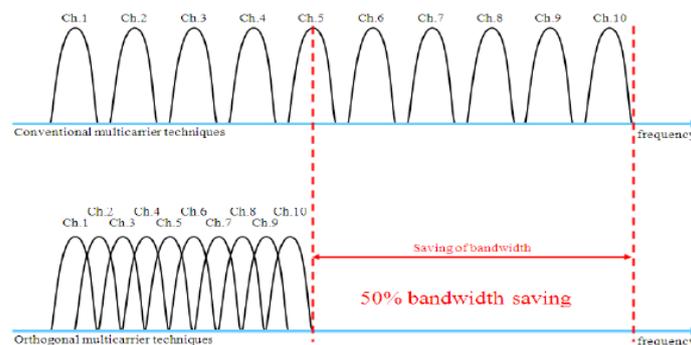


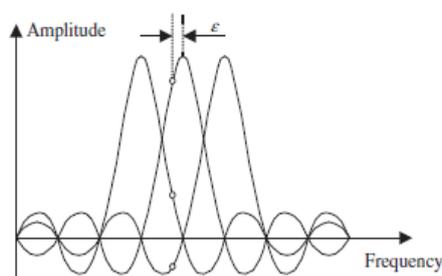
Fig.1. Bandwidth saving in OFDM

In OFDM, overlapping multicarrier modulation is used by which almost 50% saving in the bandwidth can be obtained. Fig.1 shows the above principle.

### II. Modulation Demodulation

The In OFDM, N-point IFFT and FFT are used to implement OFDM modulation and demodulation respectively. OFDM transmitter maps the message bits into a sequence of PSK or QAM symbols, which will be subsequently converted into N parallel streams. Each of N symbols from serial-to-parallel (S/P) converter is modulated on the different sub-carriers. OFDM is an effective technique to combat multipath fading. It is less sensitive to the timing errors. On the other hand, it is highly sensitive to frequency offset and phase noise. The major drawback is high peak to average power ratio (PAPR), which tends to reduce the power efficiency of high power amplifier used in OFDM transmitter. The major parameters that affect the performance of OFDM are CFO and PAPR.

In case of OFDM, the essential feature is the strict orthogonality among the sub-carriers. In case the orthogonality is not sufficiently warranted by any means, its performance may be degraded due to inter-symbol interference (ISI) and inter-channel interference (ICI) [2]. OFDM modulation has been chosen for Cellular radio, DSL & ADSL modems, DAB (Digital Audio Broadcasting) radio, DVB-T (Terrestrial Digital Video Broadcasting), HDTV broadcasting, HYPERLAN/2 (High Performance Local Area Network standard), the wireless networking standard IEEE 802.11, 4G cellular mobile system and many other broadband applications[8].



**Fig.2.** Inter-carrier interference(ICI)

Figure.2 shows that the frequency shift in the frequency-domain signal  $X[k]$  is subjected to the CFO of  $\epsilon$  and leads to an inter-carrier interference (ICI), which means any subcarrier frequency component can be affected by other subcarrier frequency components. CFO also increases the bit error rate (BER) and degrades SNR of the signal. CFO is caused by Doppler shift  $f_d$ . Let  $f_c$  and  $f_c'$  denote the carrier frequencies in the transmitter and receiver respectively. Even if we intend to generate exactly the same carrier frequencies in the transmitter and receiver, there may be an unavoidable difference between them due to the physically inherent nature of the oscillators.

Let  $f_{\text{offset}} = f_c - f_c'$  Doppler frequency  $= f_d = \frac{v \cdot f_c}{c}$   $f_c$  is the carrier frequency  $v$  velocity of receiver  $c$  is speed of light normalized CFO  $\epsilon = \frac{f_{\text{offset}}}{\Delta f}$   $\Delta f$  is subcarrier spacing  $f_{\text{offset}}$  is CFO. For CFO estimation in the time domain, cyclic prefix (CP) or training sequence is used. In frequency domain, pilot tones can be used.

### III. CFO Estimation

The study of CFO and its analysis has been carried out using both time and frequency domain. Many synchronization methods are worked upon. In Cyclic Prefix, a CFO of  $\epsilon$  results in a phase inserted in the frequency domain and transmitted in every OFDM symbol rotation of  $2\pi n/N$  in the received signal [7]. The CFO can be found from the phase angle of the product of CP and the corresponding rear part of an OFDM symbol as given by the equation

$$\hat{\epsilon} = \frac{1}{2\pi} \arg\{\sum_{n=-N_G}^{-1} y_1^*[n]y_1[n+N]\} \tag{1}$$

$y_1^*[n]y_1[n+N]$  becomes real only when there is no frequency offset.

Imaginary part of  $y_1^*[n]y_1[n+N]$  is used as CFO estimation. Since the argument operation  $\arg()$  is performed by using  $\tan^{-1}()$  the range of CFO estimation is  $[-\pi + \pi]/2\pi = [-0.5, +0.5]$  that is  $|\hat{\epsilon}| \leq 0.5$ . If two identical training symbols are transmitted consecutively, the corresponding signals with CFO of  $\epsilon$  are related with each other as follows:

$$\hat{\epsilon} = \frac{1}{2\pi} \tan^{-1}\left\{\frac{\sum_{k=0}^{N-1} \text{Im}[Y_1^*[k]Y_2[k]]}{\sum_{k=0}^{N-1} \text{Re}[Y_1^*[k]Y_2[k]}}\right\} \tag{2}$$

The basis of this technique, proposed by P.H. Moose [5], is that same data frame is repeated and the phase value of each carrier between consecutive symbols is compared. The limit for accurate estimation by Equation is  $|\epsilon| \leq 0.5$ . The range of CFO estimation can be increased  $D$  times by using training symbol with  $D$  repetitive patterns[1]. Let a transmitter send the training symbols with  $D$  repetitive patterns in the time domain, which can be generated by taking the IFFT of a pilot arrangement in the frequency domain [3].

$$D = \frac{\text{OFDM symbol length}}{\text{length of repetitive pattern}}$$

A receiver can make CFO estimation as

$$\hat{\epsilon} = \arg\left\{\sum_{n=0}^{N/D} y_1^*[n]y_1[n+N/D]\right\} \tag{3}$$

The CFO estimation range covered by this technique is  $\{|\epsilon| \leq D/2\}$  which becomes wider as  $D$  increases. Increase in estimation range is obtained at the sacrifice of mean square error performance.

The third method is that pilot tones can be inserted in the frequency domain and transmitted in every OFDM symbol for CFO tracking [6]. After estimating CFO from pilot tones in the frequency domain, the signal is compensated with the estimated CFO in the time domain. The known symbols are evenly spaced among OFDM sub channels. Fig.3 illustrates the method [7].

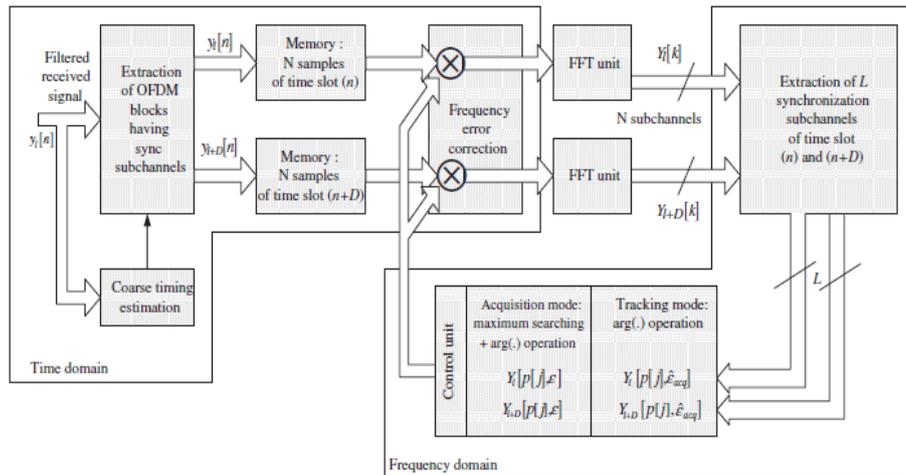


Fig.3.CFO synchronization using pilot tones

In this method, the two symbols  $y_l[n]$  and  $y_{l+D}[n]$  are saved in the memory after synchronization. Then, the signals are transformed via FFT, from which pilot tones are extracted. After estimating CFO from pilot tones in the frequency domain, the signal is compensated with the estimated CFO in the time domain. In this process, two different estimation modes for CFO estimation are implemented viz. acquisition and tracking modes. In the acquisition mode, a large range of CFO including an integer CFO is estimated. In the tracking mode, only fine CFO is estimated. The integer CFO is estimated by

$$\hat{\epsilon} = \left( \frac{1}{2\pi T_{\text{sub}}} \right) \max(\epsilon) \{ [\sum_{j=0}^{L-1} Y_{l+D}[p[j], \epsilon] Y_l^*[p[j], \epsilon] X_{l+D}^*[p[j]] X_l[p[j]]] \} \quad (4)$$

Where  $L$ ,  $p[j]$  and  $X_l[p[j]]$  denote the number of pilot tones, the location of the  $j^{\text{th}}$  pilot tone, and the pilot tone located at  $p[j]$  in the frequency domain at the  $l^{\text{th}}$  symbol period, respectively. The fine CFO is estimated by

$$\hat{\epsilon}_f = \frac{1}{2\pi T_{\text{sub}} D} \arg \{ [\sum_{j=1}^{L-1} Y_{l+D}[p[j], \epsilon_{\text{acq}}] Y_l^*[p[j], \epsilon_{\text{acq}}] X_{l+D}^*[p[j]] X_l[p[j]]] \} \quad (5)$$

The three methods CP, Moose and Classen are compared for the parameters SNR and Mean Square error. It is seen from the plot that the pilot tone based estimation performs the best because of its lowest MSE. It is found that the mean squared CFO estimation errors decrease as the SNR of received signal increases [7].

#### IV. Simulation Results

CFO estimation is done using CP, Moose and Pilot based methods.

Following parameters are used: CFO = 0.15, FFT size  $N = 128$ , Method of modulation – QAM

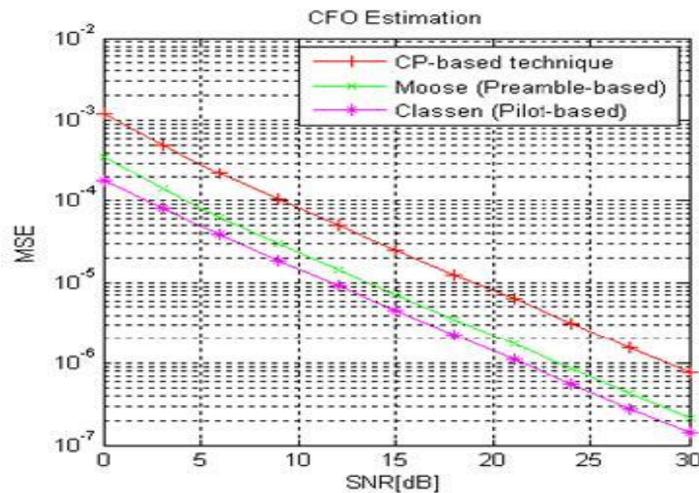


Fig.4.Comparison between CFO estimation methods

Now the value of CFO=0.30 and rest of the parameters are kept same.

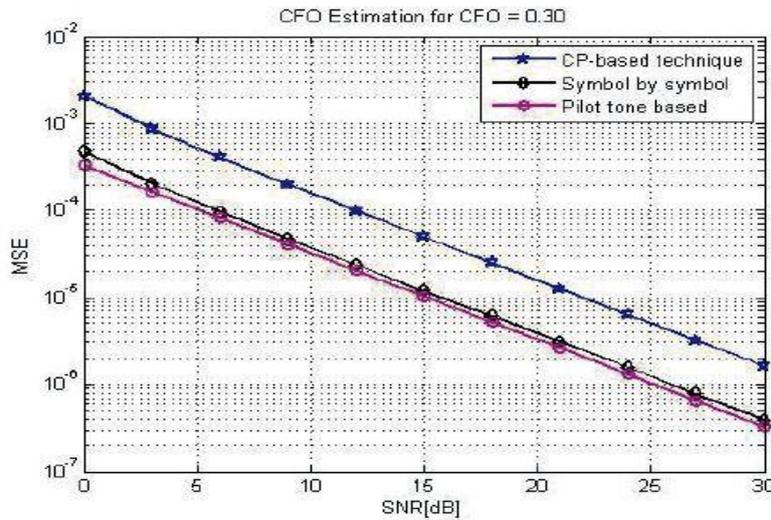


Fig.5. Comparison between CFO estimation methods.

The MSE performance for three techniques for values of CFO to be 0.30 and 0.15 is plotted. Fig.4 and Fig.5 show that pilot tone based estimation is better than the other two. The performance will vary depending upon the number of samples in CP, number of samples in preamble and number of pilot tones. The analysis of bit error rate and SNR with variations in the CFO values is done where the values of CFO vary between 0 to 0.2

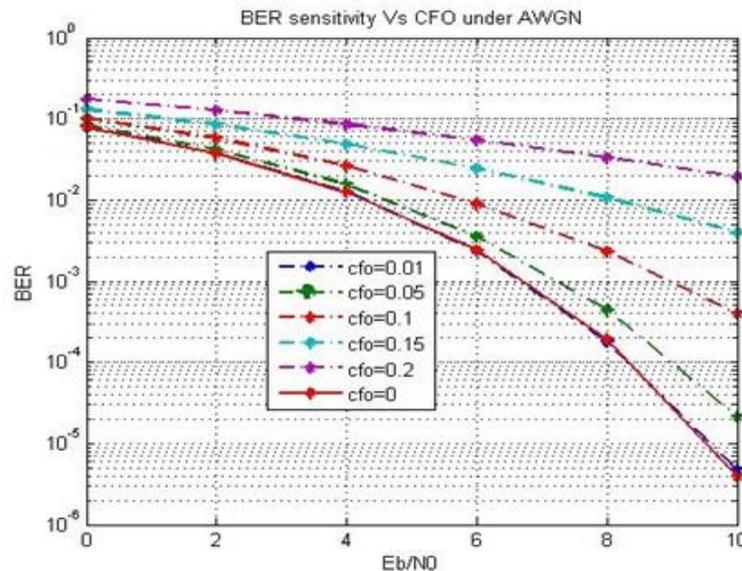


Fig.6. BER sensitivity versus CFO.

Fig.6 shows that the performance of OFDM will degrade with increasing the ratio of the maximum frequency offset to carrier spacing (i.e. CFO). The system can maintain the performance in the presence of smaller CFO. It is seen from figure.6 that increase in SNR( Eb/No) reduces Bit error rate. Now the value of CFO is varied from 0 to 0.3 keeping rest of the parameters same. It is seen from fig.7 that the performance still degrades and the curve shows a flat response.

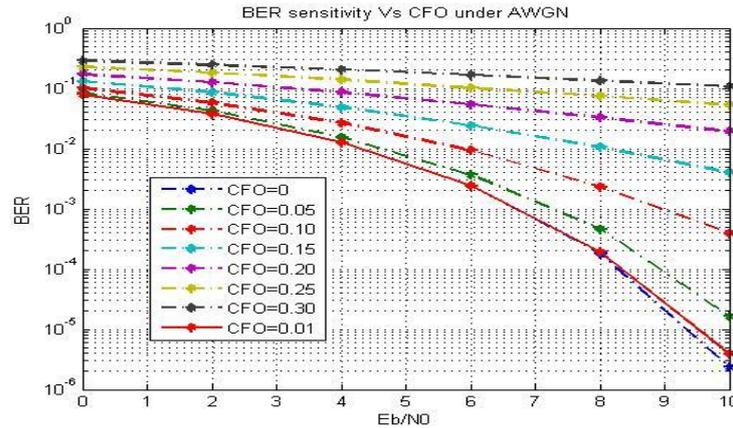


Fig.7. BER sensitivity versus CFO

The degradation of SNR, Dfreq, caused by the frequency offset is approximated by the equation

$$D_{\text{freq}} \cong \frac{10}{3 \ln 10} (\pi \Delta f T)^2 \frac{E_b}{N_0} \tag{6}$$

Where  $\Delta f$  is the frequency offset,  $T$  is the symbol duration in seconds,  $E_b$  is the energy per bit of the OFDM signal and  $N_0$  is the one-sided noise power spectrum density (PSD) [4]. The SNR degradation as a function of frequency offset is investigated. The SNR values 1, 5, 8 and 10 dB are considered. The frequency offset is kept between the range 0 to 0.5. Now the frequency offset is kept between 0 to 0.4. Fig.8 and Fig.9 shows SNR degradation is bigger for larger SNR values.

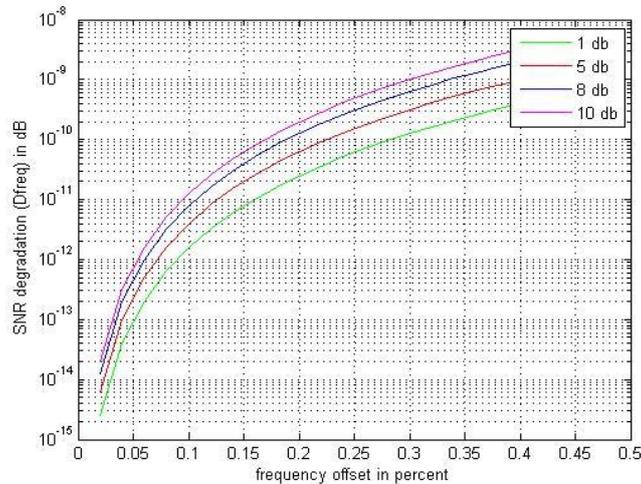


Fig.8. SNR degradation as a function of CFO

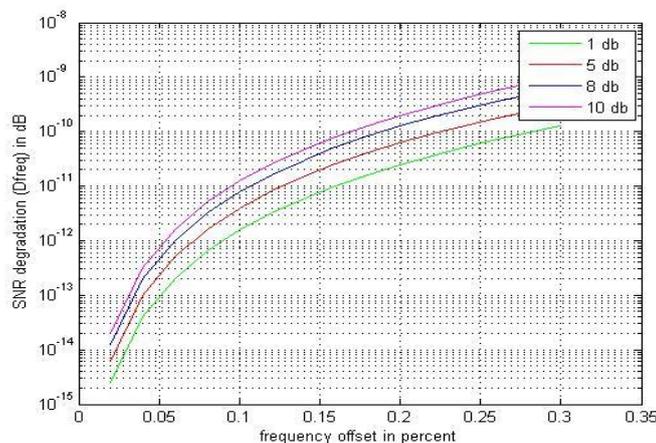


Fig.9. SNR degradation as a function of CFO

The OFDM system parameters are CFO = 0.15 and, N = 128, Ng = 16, Nps = 4 (Pilot spacing), Number of pilots Np = 32, signal to noise ratio (SNR) 0 to 25 dB, D is an integer. D=1,2,4, For OFDM mapping, QAM modulation is used. Signal energy Es = 1.

Fig.10 shows that MSE performance decreases with increasing the value of D[3].

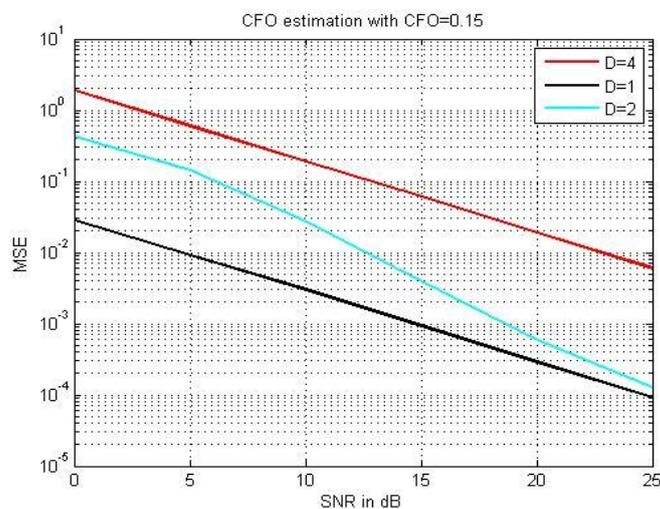


Fig.10. Training sequence based estimation.

Now the modulation was changed to QPSK all the parameters are kept constant.

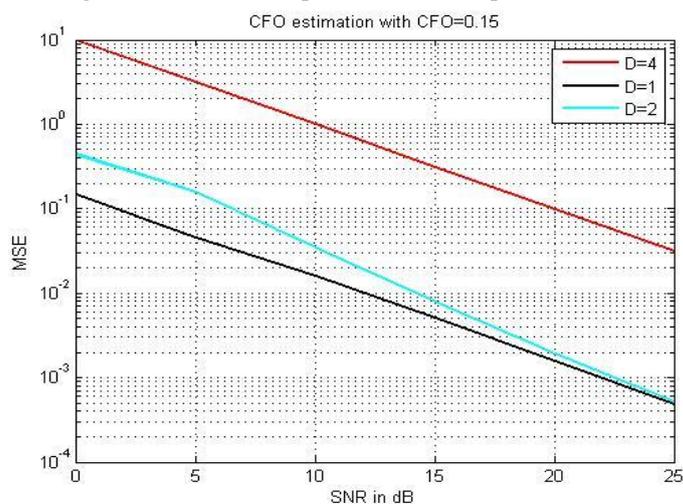


Fig.11. Training sequence based estimation.

Fig.11 shows that the mean square error is increased in case of QPSK modulation. As far as computational overhead is concerned, it is observed that the computation time required for QPSK is more than QAM.

### V. Conclusion

The importance of the study of the carrier frequency offset estimation and the source of the CFO for OFDM systems has been reviewed. OFDM is best for higher data rate applications. But the problems such as CFO require more attention. CFO introduces ICI, decreases the SNR of the signal and increases the bit error rate. Pilot tone based estimation is better than CP and preamble based estimation. The mean squared CFO estimation errors decrease as the SNR of received signal increases. In the proposed modifications the parameters were varied and the results were verified using simulations.

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